

Proposal Rights and Political Power

Tasos Kalandrakis University of Rochester

In a canonical model of sequential collective bargaining over a divisible good we show that equilibrium expected payoffs are not restricted by players' voting rights or their impatience. For all monotonic voting rules and discount factors, and for all divisions of the good among players, there exists a stationary proposal-making rule such that this division represents players' expected payoffs in a Stationary Subgame Perfect Nash equilibrium in pure strategies. The result highlights the significance of proposal rights in determining political power in collective deliberations.

The purpose of this article is to further our understanding of *political power* in a committee environment. We consider the problem of a group of two or more agents who must split a fixed divisible good. Agents cannot use physical force to divide this good. Our goal is to clarify the role and *relative significance* of other features of the political environment such as voting rights, proposal rights, agents' patience, etc. in determining political power.

Following Shapley and Shubik (1954), we gauge political power in terms of expected outcomes, rather than a priori equate it with individual prerogatives.¹ In that spirit, political power is the share of the divisible good that agents can expect to obtain when their behavior conforms to an appropriately defined equilibrium notion.²

We model committee interaction using a canonical sequential bargaining game. In each period, a member of the committee is recognized with some probability that is fixed across periods and proposes a division of the good. Committee members then vote on the proposal. If the proposal is approved in accordance with the underlying voting rule, the proposed division is implemented and the

game ends. Otherwise, the game moves to the next period with a new proposal and voting round.

This model is canonical in the sense that it follows (in basic structure) standard formulations in the bargaining literature as in Stahl (1972), Romer and Rosenthal (1978), Rubinstein (1982), Binmore (1987), Baron and Ferejohn (1989), Harrington (1990), Banks and Duggan (2000), etc. This approach parsimoniously incorporates the two types of elementary actions (proposing and voting) that are required on the part of the participants in a committee in order for this committee to "choose" an agreement.

Customary intuition, replete in both colloquial conceptions about political rights as well as in sophisticated democratic theory, suggests that political power depends on committee members' voting rights. Perhaps the starkest manifestation of the strength of that intuition in the formal literature are the various power indices arising from cooperative game theory (e.g., Shapley and Shubik 1954; Banzhaf 1965, etc.). In these formulations voting rights *determine* power. But, power indices notwithstanding, a wealth of other forces including players' patience and proposal rights also influence political power.³

Tasos Kalandrakis is assistant professor of political science, University of Rochester, Harkness Hall 333, Rochester, NY 14627-0146 (akalandr@troi.cc.rochester.edu).

Thanks to Scott Ashworth, John Duggan, Hein Goemans, Jim Johnson, Maria Montero, David Primo, and an anonymous referee for their feedback. I am responsible for any errors.

¹For review and discussion of various conceptions of political power from both positive and normative perspectives see, among others, Dahl (1957), Bachrach and Baratz (1962), Riker (1964), Gaventa (1980), and Dowding (1996).

²We recognize that this definition of power may strike some readers as narrow. Nevertheless, we are studying an environment with a very precise set of social outcomes (divisions of a good), with actors that have simple and clear objectives, all of which are common knowledge. We believe this environment leaves little room for ambiguity in defining power compared to a general social setting. One may conjecture that, if applied to the social environment we study, alternative definitions in the literature converge to the definition used in this analysis. Indeed, Wittman (1976) makes the point that ostensibly diverging definitions of power are more similar than they appear.

³In infinite period sequential bargaining games, such comparative statistics can be traced to Rubinstein (1982). Eraslan (2002) performs this analysis in the Baron and Ferejohn (1989) model.

American Journal of Political Science, Vol. 50, No. 2, April 2006, Pp. 441–448

©2006, Midwest Political Science Association

ISSN 0092-5853

Indeed, political thinking from diverse fields of study posits that rules for the origination of proposals can significantly affect political power. Our goal is to understand *how much?* Specifically, we ask whether voting rights, impatience, or the use of simple strategies place any restriction on political power *if* proposal rights can be assigned without constraint among players?

We answer this question in the negative by showing that *all* possible power levels can be obtained in equilibrium using proposal-making rules. For any monotonic voting rule and any discount factors, *every* level of power can be obtained in a Stationary Subgame Perfect (SSP) Nash equilibrium by appropriate choice of proposal probabilities.⁴ Furthermore, these equilibria are restricted to *pure proposal strategies* and voting strategies that are *stage-undominated* (Baron and Kalai 1993). The converse is not true, i.e., fixing proposal probabilities, we cannot in general obtain all power levels by independently manipulating voting rights, or discount factors, or by allowing more complex (but sequentially rational) strategies.

We emphasize that we restrict the analysis to equilibria in which agents use *pure stationary strategies*. This restriction significantly strengthens our result. Political power we can obtain using such simple strategies can also be obtained by allowing history-dependent and/or mixed strategies. In fact, the range of political power levels that can be obtained in equilibrium expands radically if committee members can devise complex strategies, as folk-theorem-like results regarding the set of Subgame Perfect Nash equilibria of related bargaining games demonstrate.^{5,6}

The use of stationary strategies that we require qualifies the associated equilibrium and strategies as *least complex* or *costly*, whether we measure complexity costs in terms of the number of states required to describe the strategy of an automaton (as in Baron and Kalai 1993), or in terms of past information used to determine actions (as in Chatterjee and Sabourian 2000). The additional restriction to pure strategies further reduces complexity costs, while, for example, Baron and Kalai (1993) perform their

analysis ignoring complexity arising from the fact that players randomize when proposing.⁷

In summary, our analysis provides a stark demonstration of the claim that proposal-making rules are a more potent source of power than voting rights, impatience, or complex equilibrium strategies. Indeed, there are no limitations on the range of power levels that can be attained by manipulation of proposal-making rules in the bargaining environment we consider. Thus, analysts, practitioners, and constitutional engineers who aim to strike a certain balance of political power may be significantly misguided by focusing exclusively⁸ on voting rights: if one must limit institutional manipulation to one aspect of the committee environment, then rules for the origination of proposals appear to be the dimension of choice.

Our analysis provides direct insight on the source of power of committee leaders. In particular, casual observation corroborates the claim that committee chairs hold considerable power. Indeed, descriptions of committee proceedings abound with anecdotes highlighting this potential influence. Yet, in formal committee settings the chair has no control over the voting rule (which is typically fixed) and is not expected/allowed to make proposals. In some settings, the chair does not even have voting rights, except in situations of a tie. The significant prerogative of the chair is the ability to determine access to proposal making by other committee members by determining recognition. Our analysis demonstrates that this ability to manipulate the proposal-making process is sufficient to bestow considerable power, despite the fact that a chair's rights as a committee member are otherwise significantly restricted.⁹

Before we move to the main analysis, we review additional related literature. Besides cooperative power indices, specific relations between power and voting rules have also been derived using noncooperative analysis. For example, Winter (1996) specifies a sequential bargaining game in which veto players have all power. Snyder, Ting, and Ansolabehere (2005) study bargaining using a class of voting rules represented by weighted majority voting. Under certain parametric restrictions they derive a power

⁴A germ of this result appears in Kalandrakis (2004), who (allowing for mixed strategies) shows that all levels of power between big and small states represented in a bicameral legislature can be obtained by appropriate assignment of proposal rights.

⁵Such theorems require patience and at least three players under unanimity (e.g., Merlo and Wilson 1995). Five or more players have been shown to be sufficient with majority rule (Baron and Ferejohn 1989).

⁶On the other hand, Kalandrakis (2006) shows that, for almost all discount factors, there exist only a finite number of SSP equilibria in pure strategies in the game we analyze.

⁷See Baron and Kalai (1993, 293), first full paragraph. They study symmetric majority rule games and show that the equilibrium in Baron and Ferejohn (1989) is the simplest.

⁸Examples of such focus on voting rules include the application of the Banzhaf or Shapley-Shubik indices in legal or similar proceedings (Dubey and Shapley 1979), or the application of power indices in various phases of constitutional design in the European Union (EU). See Holler and Widgren (1999), Lane and Berg (1999), the references therein, and in Garrett and Tsebelis (1999a), for a list of such applications.

⁹I thank an anonymous referee for suggesting this interpretation.

index to the effect that political power is *equal* to voting weights (“except for corner solutions”).

These results rely on implicit or explicit restrictions on players’ proposal rights. In the case of cooperative analyses, this becomes obvious by the fact that only special proposal-making procedures implement the underlying axiomatic solutions on which some of the power indices are premised.¹⁰ Winter (1996) and McCarty (2000a,b) explicitly discuss how assumptions about proposal making among veto and nonveto players affect power levels. Similarly, the power index of Snyder, Ting, and Ansolabehere (2005) does not hold under all possible assumptions about players’ proposal prerogatives.

There also exists an extensive literature that, in consonance with our results, emphasizes the role of proposal making in determining political power. Particularly influential in that regard has been the work of McKelvey (1976, 1979) and Romer and Rosenthal (1978). From a more normative angle, Dahl (1991), highlights agenda setting and its potential influence on decision making. Garrett and Tsebelis (1996), Tsebelis and Garrett (1996), and Garrett and Tsebelis (1999a,b) criticize the use of power indices in EU studies emphasizing, among other things, the fact that they do not account for proposal rules. Agenda-setting power—its origins, assignment, control, and consequences—figures prominently in Congressional studies on committees and proposal rules, seniority, political parties, etc.¹¹

The remainder of this article proceeds as follows. We describe the bargaining model and give a formal definition of political power for this model in the next section, then state the main result, and finally conclude.

Bargaining Model

Consider a set of $n \geq 2$ players $N = \{1, \dots, n\}$. They convene in periods $t = 1, 2, \dots$ to divide a fixed divisible good. Thus, possible agreements are denoted by a vector $\mathbf{x} = (x_1, \dots, x_n)$ drawn from a set $\Delta^n \equiv \{\mathbf{x} \in \mathbf{R}^n : \sum_{i=1}^n x_i = 1, x_i \geq 0\}$, the $(n - 1)$ -dimensional unit simplex in \mathbf{R}^n .

¹⁰e.g., Gul (1989) Hart and Mas-Colell (1996) discuss non-cooperative implementation of the Shapley value (Shapley 1953) on which the Shapley-Shubik power index is founded. Shapley and Dubey (1979) offer an axiomatic foundation for the Banzhaf index.

¹¹The literature in this area—with implicit or explicit reference to proposal prerogatives—is vast. We mention Shepsle (1979), Shepsle and Weingast (1981), Denzau and McKay (1983), Shepsle and Weingast (1987), Gilligan and Krehbiel (1987), Weingast and Marshal (1988), Krehbiel (1991), Rohde (1991), McKelvey and Riezman (1992), Cox and McCubbins (1993), Tsebelis (1994), Aldrich (1995), and Dion and Huber (1996), etc.

An agreement requires the approval of a winning coalition, $C \subseteq N$. The set of winning coalitions is determined by the underlying voting rule and is represented by a subset, \mathcal{D} , of the set of all possible subsets of N , i.e., $\mathcal{D} \subseteq 2^N$. We assume that \mathcal{D} is nonempty and *monotonic*, so that for any two coalitions A, B with $A \subseteq B \subseteq N$, we have $A \in \mathcal{D} \Rightarrow B \in \mathcal{D}$. Monotonicity requires that coalitions that encompass winning coalitions are also winning coalitions.¹² For our subsequent arguments, monotonicity of the voting rule ensures that a player that prefers to have a proposal rejected cannot do so by voting *yes*.¹³ Thus, the voting strategies we will specify constitute best responses with monotonic voting rules.

Subsumed in the class of voting rules we consider are familiar quota rules, as well as the weighted majority voting rules analyzed by Snyder, Ting, and Ansolabehere (2005). Also covered are the voting rules considered by Winter (1996), McCarty (2000a,b), as well as the two versions of bicameralism analyzed by Diermeier and Myerson (1999), and Kalandrakis (2004).¹⁴ Still, these studies represent only a small fraction of the possibilities permitted by our assumptions on the voting rule \mathcal{D} .

For any admissible voting rule, the interaction of players proceeds as follows. In each period $t = 1, 2, \dots$, prior to reaching an agreement, one of the players is recognized with probability π_i to make a proposal $\mathbf{z} \in \Delta^n$. Having observed the proposal, players vote *yes* or *no*. If a winning coalition vote *yes*, then the game ends with \mathbf{z} being implemented, else the game moves to the next period. Naturally, the vector of probabilities of recognition, $\boldsymbol{\pi}$, is such that $\boldsymbol{\pi} \in \Delta^n$.

Each player $i \in N$ derives von Neuman-Morgenstern stage utility $u_i(\mathbf{x}) = x_i$. Players discount the future by a factor $\delta_i \in [0, 1]$, $i \in N$. Thus, the payoff of $i \in N$ from a decision $\mathbf{x} \in \Delta^n$ reached in period $t \geq 1$ is given by $\delta_i^{t-1} x_i$. In the event of perpetual disagreement players receive

¹²In effect our assumptions allow the entire class of simple games and is as general, when it comes to admissible voting rules, as Banks and Duggan (2000).

¹³For example, assume $N = \{1, 2, 3\}$ and the nonmonotonic voting rule $\mathcal{D} = \{\{1, 2\}\}$, i.e., coalition $\{1, 2, 3\}$ is not winning. Now, if players 1 and 2 vote *yes* and 3 votes *no*, a proposal passes, but if all three players vote *yes* the proposal is rejected.

¹⁴McCarty and Diermeier, and Myerson analyze models of Bicameralism in which a separate allocation of the resource is made to each of the members of the two houses, while the division of the resource is, effectively, among regions (big and small) with representation in both houses in Kalandrakis. A related model of bicameralism studied by Ansolabehere, Snyder, and Ting (2003), cannot be represented in our framework because of their assumption that the allocation of the resource is among players (districts) in the lower house, while players in the upper house (states) have preferences that only depend on the size of the median allocation among districts within their state.

zero. We denote a game with voting rule \mathcal{D} , discount factors $\delta \in [0, 1]^n$, and recognition probabilities $\pi \in \Delta^n$, by $\Gamma(N, \mathcal{D}, \delta, \pi)$.

A stationary proposal strategy for player $i \in N$ in game $\Gamma(N, \mathcal{D}, \delta, \pi)$ is a division of the good, $\mathbf{z}^i \in \Delta^n$, proposed by player i when i is recognized. A stationary voting strategy is a set $A_i \subseteq \Delta^n$ that contains the divisions of the dollar on which player i votes *yes*. With stationary voting strategies $\{A_i\}_{i \in N}$, the set of acceptable divisions in each period is given by $A = \cup_{C \in \mathcal{D}} (\cap_{h \in C} A_h)$. We say that strategies $\{\mathbf{z}^i, A_i\}_{i \in N}$ involve *no-delay* if $\mathbf{z}^i \in A$ for each $i \in N$.

Given stationary strategies for all players, the continuation value of $i \in N$, denoted by v_i , is defined as the expected utility of i if the game moves in the next period. A *no-delay, pure strategy Stationary Subgame Perfect (SSP) Nash equilibrium* for game $\Gamma(N, \mathcal{D}, \delta, \pi)$ is a set of stationary strategies $\{\mathbf{z}^i, A_i\}_{i \in N}$ and a corresponding vector of continuation values $\mathbf{v} \in \Delta^n$, such that for all $i \in N$, we have $v_i = \sum_{h \in N} \pi_h z_i^h$, $\mathbf{x} \in A_i \Leftrightarrow x_i \geq \delta_i v_i$, and $\mathbf{z}^i \in \arg \max\{x_i : \mathbf{x} \in A\}$.

In what follows we will refer to such an equilibrium simply as a pure strategy SSP equilibrium, omitting the no-delay property. Note that we require that players vote *yes* if and only if they prefer the proposed agreement to their discounted continuation value, $\delta_i v_i$. This rules out implausible SSP equilibria in which players vote in arbitrary ways on proposed agreements due to the fact they are not pivotal. Baron and Kalai (1993) refer to these refined voting strategies as *stage undominated*.

We call equilibrium continuation value v_i the *political power* of player i in game $\Gamma(N, \mathcal{D}, \delta, \pi)$, since it corresponds to that player's *ex ante* expected fraction of the good. For clarity, we restate this as a formal definition:

Definition 1 $\mathbf{v} \in \Delta^n$ is a political power vector for players $1, \dots, n$ in game $\Gamma(N, \mathcal{D}, \delta, \pi)$ if it represents players' continuation values in a SSP Nash equilibrium.

Existence of SSP Nash equilibrium is not an issue for game $\Gamma(N, \mathcal{D}, \delta, \pi)$ by the arguments of Banks and Duggan (2000). We emphasize that an SSP equilibrium in this game is both Nash and Subgame Perfect. In particular, if all players other than $i \in N$ play a stationary strategy, then i has a stationary best response: i can find a stationary strategy such that there does not exist another strategy (stationary or not) that can improve her payoff. In other words, players are allowed to contemplate deviations to nonstationary strategies, yet such strategies cannot be better responses.

Note that we do not rule out the existence of multiple political power vectors for the same game, although

uniqueness does obtain in the case of simple majority rule (Eraslan 2002). The set of possible levels of political power, $\mathbf{v} = (v_1, \dots, v_n)$, in game $\Gamma(N, \mathcal{D}, \delta, \pi)$ is the set of all possible divisions of the good, Δ^n . In the next section we shall show that for every voting rule \mathcal{D} , and any discount factors $\delta \in [0, 1]^n$, every possible political power $\mathbf{v} \in \Delta^n$ can be obtained in game $\Gamma(N, \mathcal{D}, \delta, \pi)$ by appropriate choice of recognition probabilities $\pi \in \Delta^n$.

Indeterminacy of Power

We start this section with a pair of examples in which we illustrate how prespecified levels of power can be obtained by appropriate choice of recognition probabilities. These examples pave the way for the main result of our analysis which we state toward the end of this section.

We first consider a four-player committee with a member that has veto over decisions:

Example 1 (Committee with Outside Veto) Assume $n = 4$ and $\delta_i = \frac{1}{2}$ for all $i \in N$. Player 4 can veto decisions and a majority of the remaining players must also approve proposals. Thus

$$\mathcal{D} = \{\{4, 1, 2\}, \{4, 1, 3\}, \{4, 2, 3\}, N\}.$$

Consider the egalitarian value $\mathbf{v}^* = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. For this \mathbf{v}^* (and assumed discount factors), $\mathbf{z}^1 = (\frac{3}{4}, 0, \frac{1}{8}, \frac{1}{8})$, $\mathbf{z}^2 = (0, \frac{3}{4}, \frac{1}{8}, \frac{1}{8})$, $\mathbf{z}^3 = (0, \frac{1}{8}, \frac{3}{4}, \frac{1}{8})$, and $\mathbf{z}^4 = (\frac{1}{8}, \frac{1}{8}, 0, \frac{3}{4})$ constitute optimal proposals for the respective players. In particular, all proposals obtain the vote of the veto player 4 and a majority of two from the remaining three players. Using these proposals and \mathbf{v}^* we formulate equations $v_i^* = \sum_{h=1}^4 \pi_h z_i^h$, $i = 1, \dots, 4$, and $\sum_{i=1}^4 \pi_i = 1$. For recognition probabilities $\pi_1^* = \frac{3}{10}$, $\pi_2^* = \frac{6}{25}$, $\pi_3^* = \frac{13}{50}$, and $\pi_4^* = \frac{1}{5}$ these equations are satisfied. Thus, the proposals form an SSP equilibrium and \mathbf{v}^* represents players' political power in the associated game.

Note that the veto player receives a payoff that is exactly equal to that of the remaining players. In effect, this requires that the veto player has a smaller probability of making proposals. Next, consider a dictatorial voting rule:

Example 2 (Dictatorial Rule) Assume $n = 3$, $\delta_1 = \frac{3}{4}$, and $\delta_2 = \delta_3 = \frac{1}{4}$. Let player 1 be the dictator so that

$$\mathcal{D} = \{\{1\}, \{1, 2\}, \{1, 3\}, N\}$$

Consider the value $\mathbf{v}^* = (\frac{1}{9}, \frac{4}{9}, \frac{4}{9})$. For this \mathbf{v}^* (and the assumed discount factors), $\mathbf{z}^1 = (1, 0, 0)$, $\mathbf{z}^2 = (\frac{1}{12}, \frac{11}{12}, 0)$, and $\mathbf{z}^3 = (\frac{1}{12}, 0, \frac{11}{12})$ constitute optimal proposals since players 2 and 3 must obtain the consent of the dictatorial voter, while player 1 can dictate the outcome. We now formulate

equations $v_i^* = \sum_{h=1}^3 \pi_h z_i^h$, $i = 1, 2, 3$, and $\sum_{i=1}^3 \pi_i = 1$, and obtain the solution $\pi_1^* = \frac{1}{33}$, and $\pi_2^* = \pi_3^* = \frac{16}{33}$.

In the above example we have “stacked the deck” in favor of player 1. Not only does player 1 have dictatorial voting power, but she is also more patient ($\delta_1 > \delta_2 = \delta_3$) than the other two players. As a result, her bargaining power is higher, *all else equal*. But when all else is not equal, in particular when players 2 and 3 have significantly higher proposal-making probability, their political power can be larger than that of the dictatorial voter.

We could proceed to derive similar outcomes for other voting rules or sustain alternative equilibrium values in the above cases. We trust that these examples provide a stark illustration of our argument. With appropriate choice of proposal rights, even players that have no voting rights (i.e., dummy players that are superfluous in every winning coalition) such as players 2 and 3 in the last example, can have more power than a dictatorial voter.

In both examples we considered, we fixed the voting rule, \mathcal{D} , discount factors, $\delta \in [0, 1]^n$, and ‘target’ political power, $\mathbf{v}^* \in \Delta^n$, and sought n recognition probabilities, π_i , $i \in N$, for the players that satisfy a set of $n + 1$ linear equations. These probabilities *must also satisfy n inequalities*, $\pi_i \geq 0$, $i \in N$. By adopting a formulation introduced by Nash (1951), we show that a solution to these equalities and inequalities exists for every admissible \mathcal{D} , δ , and \mathbf{v}^* . Specifically:

Proposition 1 *Consider any nonempty, monotonic voting rule, \mathcal{D} , and any discount factors $\delta \in [0, 1]^n$. For every $\mathbf{v}^* \in \Delta^n$ there exists a set of recognition probabilities $\pi^* \in \Delta^n$ such that \mathbf{v}^* is the political power of game $\Gamma(N, \mathcal{D}, \delta, \pi^*)$ in a SSP Nash equilibrium in pure strategies.*

Proof. See the appendix.

It is important to emphasize what proposition 1 does not state. First, it does not imply voting rights are irrelevant. All else constant, the voting rule does affect players’ expected payoff. Similarly, the typical conclusion in sequential bargaining games regarding the effect of discounting is not threatened by proposition 1. What proposition 1 does make plain is that proposal power “trumps” these other aspects of the environment that determine players’ power.

The converse is not true in general, as we will now show (by example) for the case of voting rules, discount factors, and history dependent strategies that satisfy sequential rationality. First, consider a game with $N = \{1, 2\}$, $\pi_i = \frac{1}{2}$, and discount factors $\delta_i = \delta < 1$, $i = 1, 2$. For this game it is not possible to devise a vot-

ing rule that will sustain equilibrium value $\mathbf{v}^* = (1, 0)$. Next, for the same example under unanimity rule ($\mathcal{D} = \{N\}$), there is a unique subgame perfect Nash equilibrium, i.e., even if we allow equilibria in which players use complex strategies, there is a unique equilibrium value $\mathbf{v}^* = (\frac{1}{2}, \frac{1}{2})$. Finally, it is easy to see that in a simple majority rule game with $N = \{1, 2, 3\}$ and $\pi_1 = 1$, there is no choice of discount factors for the three players that sustains equilibrium value $\mathbf{v}^* = (0, x, 1 - x)$, $x \in [0, 1]$.¹⁵

Discussion and Conclusions

We have shown that in standard bargaining models over a fixed divisible resource, the voting rule, impatience, or the use of simple strategies do not jointly restrict political power if we can manipulate proposal rights. This result quantifies the relative significance of proposal making in influencing political power in comparison to voting rights. It also highlights the importance of proposal rights as an institutional dimension with which to influence political outcomes. We spend the next few paragraphs in an attempt to anticipate objections that can be leveled to these conclusions.

One objection to the significance of our result for the purposes of institutional design is that proposal prerogatives either cannot be manipulated via rules or that they empirically correlate with voting rights. We do not dispute proposal and voting rights correlate in many existing committee configurations. In theoretical studies such a correlation stems from the natural benchmark assumption to treat individual committee members symmetrically. But despite the empirical or theoretical relevance of restrictions on the assignment of proposal rights, the fact remains that proposal rights and voting rights can vary independently.

Importantly, constitutions can be written to determine proposal rules in arbitrary ways. In the EU, for example, small and big countries rotate with equal frequency in the presidency of the Council of ministers, even though

¹⁵It is possible (in similar spirit to the approach in examples 1 and 2) to write a system of equations in order to obtain a set of discount factors that produce given levels of power. But we cannot reach the same conclusion as in Proposition 1 regarding discount factors because the solutions of such equations do not always satisfy the inequalities $0 \leq \delta_i \leq 1$, $i \in N$. Substantively, proposal probabilities are more effective at manipulating power than discount factors because a player’s discount factor influences only the amount that player is allocated (if any) by other players. On the other hand, a proposer guarantees herself inclusion in the winning coalition, in fact extracting a residual surplus following optimal coalition building. I thank an anonymous referee for suggesting this comparison between discount factors and recognition probabilities.

the voting rule is weighted according to population. Primo (2002) discusses formal institutions operating under majority rule that stipulate a unique proposer, i.e., assign all proposal power to a single individual.

We can find similar examples when considering informal committee environments. In certain cultures, senior members are seldom interrupted when expressing opinion and, hence, enjoy privileged access to proposal making in council or committee meetings. Similarly, readers familiar with tumultuous student deliberations in amphitheatres will recognize that proposal power is markedly higher for the subset of participants that belong in the intersection of loud and eloquent speakers.

Even in otherwise delineated democratic polities such as the United States, campaign finance practices imply significantly reduced political power for the poorer segments of society that otherwise have equal voting rights. This is because access to a developed media industry via monetary contributions is essential for a viable electoral campaign and subsequently for the formulation of proposals. In short, individual heterogeneity can and often does induce a divergence between de facto or de jure proposal rights and voting rights.

A related objection has to do with the probabilistic nature of proposal prerogatives in our model. In particular, probabilistic recognition generates an infinite set of proposal making institutions to select from, while the choice of voting rules is effectively among a determinate number in finite committees. We deem probabilistic recognition a reasonable assumption in the absence of explicit rules that determine the identity of the proposer. The fundamental insight from the literature on proposal rights (and from our study) is that proposing in such settings is desirable. As a result it seems natural that the resolution of agent's competition for access to proposal making takes place randomly when no other rule or individual resolves this indeterminacy. For example, empirical evidence suggests that the selection of formateurs during government formation negotiations in certain multiparty parliamentary systems is probabilistic rather than deterministic in accordance to the size of the parties' parliamentary representation (Diermeier and Merlo 2004).

But, empirical relevance aside, the question of whether probabilistic recognition is a natural assumption is not relevant from the point of view of institutional design. There are no technological constraints that would prevent the author(s) of committee rules or constitutions to stipulate the determination of the proposer via some randomization device such as the one we model. In other words, probabilistic recognition is a feasible institutional choice (even if the resultant distribution of political power is not acceptable or politically viable).

Lastly, readers may object to the idea that committee interaction follows the rules specified in our model versus some other more elaborate sequence of proposal making and voting on the part of committee members. Certainly real-world constitutions and parliamentary rules of procedure are more complicated than what we assume. The restricted set of institutions for proposal making we consider renders our result even stronger. In particular, if we can choose from a richer class of rules and procedures for the origination of proposals compared to the one we allow, we can achieve more equilibrium power levels than those permitted under the maintained assumptions.

Admittedly, the complex parliamentary rules we encounter in real legislatures exist in part in order to accommodate informational or other organizational needs (e.g., gains from exchange) emanating from the policy space. These considerations are not relevant in the policy space we study. Another important reason for the complexity of legislative rules is direct evidence in favor of our conclusion: self-organizing legislatures adopt elaborate rules to safeguard against the abuse of proposal prerogatives exactly because proposal making can have such a significant impact on outcomes.

Appendix: Proof of Proposition 1

For each player $i \in N$ consider a winning coalition $C^i \in \mathcal{D}$ such that

$$C^i \in \arg \max \left\{ 1 - \sum_{h \in C^i \setminus \{i\}} \delta_h v_h^* : C \in \mathcal{D} \right\} \quad (1)$$

C^i satisfying (1) may not be unique in general, in which case simply select one among these 'optimal' coalitions. Since $\sum_{h \in N} \delta_h v_h^* \leq 1$ for all $\mathbf{v}^* \in \Delta^n$, we can construct a proposal $\mathbf{z}^{i*} \in \Delta^n$ that corresponds to selected coalition C^i and takes the form:

$$z_j^{i*} = \begin{cases} \delta_j v_j^* & \text{if } j \in C^i \setminus \{i\} \\ 1 - \sum_{h \in C^i \setminus \{i\}} \delta_h v_h^* & \text{if } j = i \\ 0 & \text{otherwise} \end{cases}$$

If players use proposal strategies $\mathbf{z}^{i*} \in \Delta^n$, $i \in N$ and recognition probabilities are given by $\boldsymbol{\pi} \in \Delta^n$, we obtain a new vector of continuation values $\mathbf{v}(\boldsymbol{\pi}) \in \Delta^n$ according to the formula $v_i(\boldsymbol{\pi}) \equiv \sum_{h \in N} \pi_h z_i^{h*}$, $i \in N$. To prove the proposition, it suffices to find $\boldsymbol{\pi}^* \in \Delta^n$ such that $\mathbf{v}(\boldsymbol{\pi}^*) = \mathbf{v}^*$. Then, stationary strategies $\{\mathbf{z}^{i*}, A_i^*\}_{i \in N}$, where $A_i^* = \{\mathbf{x} \in \Delta^n : x_i \geq \delta_i v_i^*\}$ form a SSP equilibrium with political power $\mathbf{v}^* \in \Delta^n$.

To find such a $\boldsymbol{\pi}^*$, we construct a continuous function $\Pi : \Delta^n \rightarrow \Delta^n$, that maps the space of recognition probabilities into itself. We define

$$\Pi_i(\boldsymbol{\pi}) \equiv \frac{\pi_i + \phi_i(\boldsymbol{\pi})}{1 + \sum_{h=1}^n \phi_h(\boldsymbol{\pi})}, \quad i \in N$$

where the continuous function $\phi_i : \Delta^n \rightarrow \mathbf{R}_+$, is given by $\phi_i(\boldsymbol{\pi}) \equiv \max\{0, v_i^* - v_i(\boldsymbol{\pi})\}$. Now Π maps the convex, compact Δ^n into itself, hence it has a fixed point $\Pi(\boldsymbol{\pi}^*) = \boldsymbol{\pi}^*$, by Brouwer's theorem. Furthermore, it is obvious that $\boldsymbol{\pi}^* = \Pi(\boldsymbol{\pi}^*)$ if and only if $\phi_i(\boldsymbol{\pi}^*) = 0$ for all $i \in N$. Thus, for the fixed point $\boldsymbol{\pi}^*$ we must have $\mathbf{v}(\boldsymbol{\pi}^*) = \mathbf{v}^*$, and the proof is complete.

References

- Aldrich, John H. 1995. *Why Parties? The Origin and Transformation of Political Parties in America*. Chicago: University of Chicago Press.
- Ansolabehere, Stephen, James Snyder, and Michael Ting. 2003. "Bargaining in Bicameral Legislatures: When and Why Does Malapportionment Matter?" *American Political Science Review* 97(3):471–81.
- Bachrach, P. and M. Baratz. 1962. "2 Faces of Power." *American Political Science Review* 56(4):947–52.
- Banks Jeffrey S., and John Duggan. 2000. "A Bargaining Model of Collective Choice." *American Political Science Review* 94(March):73–88.
- Banzhaf, J. F., III. 1965. "Weighted Voting Doesn't work: A Mathematical Analysis." *Rutgers Law Review* 19:317–43.
- Banzhaf, J. F., III. 1966. "Multi-member Electoral Districts—Do They Violate the 'One Man, One Vote' Principle?" *Yale Law Journal* 75:1309–38.
- Banzhaf, J. F., III. 1968. "One Man 3.312 Votes: A Mathematical Analysis of the Electoral College." *Villanova Law Review* 13:304–32.
- Baron, David P., and John A. Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 83(December):1181–206.
- Baron, David P., and Ehud Kalai. 1993. "The Simplest Equilibrium of a Majority Rule Game." *Journal of Economic Theory* 61:290–301.
- Binmore, Ken. 1987. "Perfect Equilibria in Bargaining Models." In *The Economics of Bargaining*, eds. Ken Binmore and Partha Dasgupta. Basil Blackwell: Oxford and New York, pp.
- Binmore, Ken, and Partha Dasgupta, eds. 1987. *The Economics of Bargaining*. Basil Blackwell: Oxford and New York.
- Chatterjee, Kalyan, and Hamid Sabourian. 2000. "Multiperson Bargaining and Strategic Complexity." *Econometrica* 68(6):1491–509.
- Cox, G. W., and Mathew D. McCubbins. 1993. *Legislative Leviathan: Party Government in the House*. Berkeley: University of California Press.
- Dahl, Robert A. 1957. "The Concept of Power." *Behavioral Sciences* 20:201–15.
- Dahl, Robert A. 1991. *Democracy and Its Critics*. New Haven: Yale University Press.
- Diermeier, D., and A. Merlo. 2004. "An Empirical Investigation of Coalitional Bargaining Procedures." *Journal of Public Economics* 88(3–4):783–97.
- Diermeier, D., and R. Myerson. 1999. "Bicameralism and Its Consequences for the Internal Organization of Legislatures." *American Economic Review* 89(5):1182–97.
- Dion, D., and J. D. Huber. 1996. "Procedural Choice and the House Committee on Rules." *Journal of Politics* 58(1):25–53.
- Dowding, Keith. 1996. *Power*. Minnesota: University of Minnesota Press.
- Dubey, Pradeep, and Lloyd S. Shapley. 1979. "Mathematical Properties of the Banzhaf Index." *Mathematics of Operations Research* 4(2):99–131.
- Eraslan, H. 2002. "Uniqueness of Stationary Equilibrium Payoffs in the Baron-Ferejohn Model." *Journal of Economic Theory* 103(1):11–30.
- Garrett, Geoffrey, and George Tsebelis. 1996. "An Institutional Critique of Intergovernmentalism." *International Organization* 50(2):269–99.
- Garrett, Geoffrey, and George Tsebelis. 1999a. "Why Resist the Temptation of Power Indices in the EU." *Journal of Theoretical Politics* 11(3):291–308.
- Garrett, Geoffrey, and George Tsebelis. 1999b. "More Reasons to Resist the Temptation of Power Indices in the EU." *Journal of Theoretical Politics* 11(3):331–38.
- Gaventa, J. 1980. *Power and Powerlessness*. London: Oxford University Press.
- Gul, Faruk. 1989. "Bargaining Foundations of Shapley Value." *Econometrica* 57(1):81–95.
- Gilligan, Thomas and Keith Krehbiel. 1987. "Collective Decision Making and Standing Committees: An informational Rationale for Restrictive Amendment Procedures." *Journal of Law, Economics, and Organization* 3(2):145–93.
- Harrington J. E. 1990. "The Power of The Proposal Maker in a Model of Endogenous Agenda Formation." *Public Choice* 64(1):1–20.
- Hart, Sergiu, and Andreu Mas-Colell. 1996. "Bargaining and Value." *Econometrica* 64(2):357–80.
- Holler, Manfred, and Mika Widgren. 1999. "Why Power Indices for Assessing European Union Decision Making?" *Journal of Theoretical Politics* 11(3):321–30.
- Kalandrakis, Tasos. 2004. "Bicameral Winning Coalitions and Equilibrium Federal Legislatures." *Legislative Studies Quarterly* XXIX(1):49–80.
- Kalandrakis, Tasos. 2006. "Regularity of Pure Strategy Equilibrium Points in a Class of Bargaining Games." *Economic Theory* 28(2):309–29.
- Krehbiel, Keith. 1991. *Information and Legislative Organization*. Ann Arbor: University of Michigan Press.
- Lane, Jan-Erik, and Sven Berg. 1999. "Relevance of Voting Power." *Journal of Theoretical Politics* 11(3):309–20.
- McCarty, Nolan. 2000a. "Presidential Pork: Executive Veto Power and Distributive Politics." *American Political Science Review* 94(March):117–30.
- McCarty, Nolan. 2000b. "Proposal Rights, Veto Rights, and Political Bargaining." *American Journal of Political Science* 44(3):506–22.
- McKelvey, Richard. 1976. "Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control." *Journal of Economic theory* 12:472–82.

- McKelvey, Richard. 1979. "General Conditions for Global Intransitivities in Formal Voting Models." *Econometrica* 47:1086–112.
- McKelvey, Richard, and Raymond Reizman. 1992. "Seniority in Legislatures." *American Political Science Review* 86(4):951–65.
- Merlo, Antonio, and Charles Wilson. 1995. "A Stochastic Model of Sequential Bargaining with Complete Information." *Econometrica* 63(2):371–99.
- Nash, John. 1951. "Non-Cooperative Games." *The Annals of Mathematics*, 2nd Ser., 54(2):286–95.
- Primo, David. 2002. "Rethinking Political Bargaining: Policy-making with a Single Proposer." *Journal of Law Economics and Organization* 18(2):411–27.
- Riker, W. H. 1964. "Some Ambiguities in the Notion of Power." *American Political Science Review* 58(2):341–49.
- Rohde, David W. 1991. *Parties and Leaders in the Postreform House*. Chicago: University of Chicago Press.
- Romer, Thomas, and Howard Rosenthal. 1978. "Political Resource Allocation, Controlled Agendas, and the Status Quo." *Public Choice* 33:27–44.
- Rubinstein, Ariel. 1982. "Perfect Equilibrium in a Bargaining Model." *Econometrica* 50:97–110.
- Shapley, L. S. 1953. "A Value for N-Person Games." *Annals of Mathematics Study* 28:307–17.
- Shapley, L. S., and M. Shubik. 1954. "A Method for Evaluating the Distribution of Power in a Committee System." *American Political Science Review* 787–92.
- Shepsle, Kenneth. 1979. "Institutional Arrangements and Equilibrium in Multidimensional Voting Models." *American Journal of Political Science* 32:27–59.
- Shepsle, Kenneth, and Barry Weingast. 1981. "Structure-Induced Equilibrium and Legislative Choice." *Public Choice* 37:503–19.
- Shepsle K.A., and B.R. Weingast. 1987. "The Institutional Foundations of Committee Power." *American Political Science Review* 81(1):85–104.
- Snyder, James, Michael Ting, and Stephen Ansolabehere. 2005. "Legislative Bargaining Under Weighted Voting." *American Economic Review* 95(4):981–1004.
- Stahl, I. 1972. *Bargaining Theory*. Stockholm: Stockholm School of Economics.
- Tsebelis, George. 1994. "The Power of the European Parliament as a Conditional Agenda Setter." *American Political Science Review* 88(1):128–42.
- Tsebelis, George, and Geoffrey Garrett. 1996. "Agenda Setting Power, Power Indices and Decision Making in the European Union." *International Review of Law and Economics* 16(3):345–62.
- Weingast, Barry R., and William Marshal. 1988. "Industrial Organization of Congress." *Journal of Political Economy* 96:132–63.
- Winter, Eyal. 1996. "Voting and Vetoing." *American Political Science Review* 90(4):813–23.
- Wittman, D. 1976. "Various Concepts of Power—Equivalence Among Ostensibly Unrelated Approaches." *British Journal of Political Science* 6:449–62.