

Private Polling in Elections and Voter Welfare

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Starting point

- How do candidates use private information generated from polling?
- How does private information affect voter welfare?
- Our answers shed light on issues of candidate separation:
 - * Why candidates separate?
 - * How much candidates separate?
 - * Do candidates separate enough?

Outline of talk

- literature
- model
- equilibrium analysis
- voter welfare

Private polling literature

- Ledyard (1989)
- Chan (2001)
- Ottaviani and Sorensen (2002)
- Bernhardt, Duggan, and Squintani (2005)

Separation literature

- Downs: MVT \Rightarrow No separation.
- Probabilistic voting with office-motivated candidates: No separation.
- Probabilistic voting with policy-motivated candidates: Separation through preferences.
- Candidate with valence advantage: Separation through mixing.
- Private polling: Separation through information.

Elections

- Two candidates, A and B .
- Candidates simultaneously adopt policy platforms, x and y , which lie in the real line.
- Voters have symmetric utilities, and the median voter is uniquely defined with ideal point μ .
- Candidate A wins if $|x - \mu| < |y - \mu|$, A loses if the reverse holds, and A ties if $|x - \mu| = |y - \mu|$ (a coin flip decides the winner).
- Each candidate seeks to maximize the probability of winning.

Information

- Median voter's location has two components: $\mu = \alpha + \beta$, where
 - * α is uniformly distributed on $[-a, a]$,
 - * β is discrete, with finite support $b_1 < \dots < b_N$.
- Candidates A and B receive real-valued signals i and j about β , while α is independently distributed.
- Stories:
 - * a component of preferences is not revealed by polling,
 - * polling occurs first, then voter preferences are subject to shocks, then the election is held.

Information (cont.)

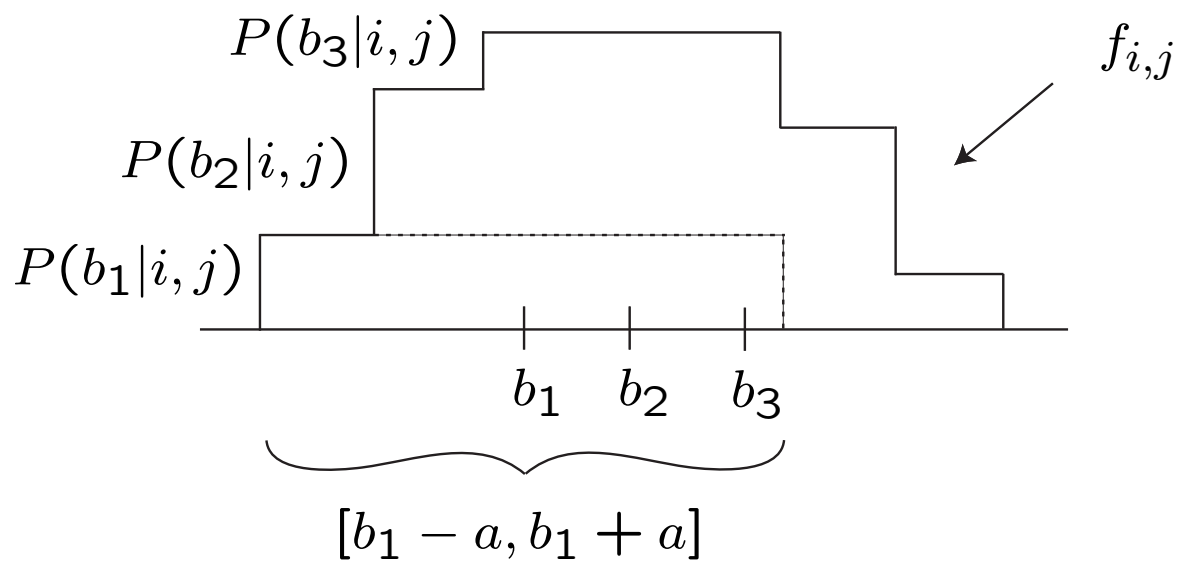
- Let $P(b, i, j)$ be the probability of (b, i, j) , with usual notation for marginals and conditionals: e.g., $P(i, j)$, $P(j|i)$.
- Self-possibility: $P(i, i) > 0$.
- Symmetry: $P(i, j) = P(j, i)$ and $F_{i,j} = F_{j,i}$.
- Arbitrary correlation is possible. Perfect correlation is Bayesian version of probabilistic voting model.

Information (cont.)

- Let $F_{i,j}$ be the distribution of the median voter conditional on signals i and j , with density $f_{i,j}$.
- Let $m_{i,j}$ denote the median of $F_{i,j}$. Note that $m_{i,j} = E[b|i, j]$.
- WLOG: $i < j$ implies $m_{i,i} < m_{j,j}$.
- Ordered signals: $i < j$ implies $m_{i,K} < m_{j,K}$.
- Sufficient uncertainty: $a > b_N - b_1$.

Information (cont.)

- A three “state” picture:



- In general,

$$F_{i,j}(z) = \frac{a - m_{i,j} + z}{2a}$$

for all $z \in [b_N - a, b_1 + a]$.

The game

- Pure strategies: $X = (x_i)$ and $Y = (y_j)$.
- Mixed strategies: $G = (G_i)$ and $H = (H_j)$.
- Ex post payoff is probability of winning:
For A , it is

$$\begin{aligned} & \Pi_A(X, Y|i, j) \\ &= \begin{cases} F_{i,j} \left(\frac{x_i + y_j}{2} \right) & \text{if } x_i < y_j \\ 1 - F_{i,j} \left(\frac{x_i + y_j}{2} \right) & \text{if } y_j < x_i \\ \frac{1}{2} & \text{if } x_i = y_j. \end{cases} \end{aligned}$$

The game (cont.)

- Ex ante payoff is

$$\Pi_A(X, Y) = \sum_{i,j} P(i, j) \Pi_A(X, Y | i, j).$$

- Note: This is a two-player, symmetric, zero-sum game.
- Solution: Bayesian equilibrium.

Bernhardt, Duggan, and Squintani (2005)

- If (X, Y) is a pure strategy equilibrium, then $x_i = y_i = m_{i,i}$ for all signals i .
- Adding an arbitrarily small amount of private information to the Downsian model can lead to non-existence of pure strategy equilibrium.
- Mixed strategy equilibria always exist.
- In models “close” to Downsian, mixed strategy equilibria “converge” to the median voter’s ideal point.

Pure strategy equilibrium

- Let $i = 0$ be such that $m_{0,0}$ equals the unconditional median. Given $i > 0$, we plausibly have $m_i < m_{i,i}$.
- With two or three signals, non-zero signals are always “self-reinforcing,” in this sense.
- Thus, candidate’s with positive signals tend to overshoot the median voter.
- (A1) Average medians:

$$m_{i,j} = \frac{m_{i,i} + m_{j,j}}{2}.$$

Pure strategy equilibrium (cont.)

- **Theorem 1.** Under (A1), a necessary and sufficient condition for existence of the only possible pure strategy equilibrium is that

$$\sum_{j:j \leq i} P(j|i) \geq \frac{1}{2}$$

and

$$\sum_{j:j \geq i} P(j|i) \geq \frac{1}{2}$$

for all signals i .

- This always holds in the two-signal model, assuming signals are not negatively correlated.
- For the extremal signals, existence requires $P(i|i) \geq \frac{1}{2}$. This is restrictive when the number of signals is large.

Mixed strategy equilibrium

- We now suppose candidates locate in a non-deterministic way.
- (A2) Stochastic dominance: For all signals i and k with $k < i$,

$$\sum_{j:j < \ell} P(j|k) \geq \sum_{j:j < \ell} P(j|i) \text{ for all } \ell.$$

- (A3) Increasing signals: For all signals i and k with $k < i$,

$$\sum_{j:j < i} P(j|i) \geq \sum_{j:j < k} P(j|k)$$

and

$$\sum_{j:j \leq i} P(j|i) \geq \sum_{j:j \leq k} P(j|k).$$

Mixed strategy equilibrium (cont.)

- Let C be the set of signals i satisfying the condition of Theorem 1. Let
- **Proposition 1.** Under (A2) and (A3), the set C is non-empty and connected.
- We refer to signals in C as “moderate.”
- A mixed strategy G is **ordered** if:
 - * for all i , $\text{Supp}G_i = [\underline{x}_i, \bar{x}_i]$,
 - * for all i and j with $i < j$, $\bar{x}_i \leq \underline{x}_j$.

Necessary conditions

- **Theorem 2.** Under (A2) and (A3), if there is an ordered equilibrium, then it is unique and has the following symmetric form.

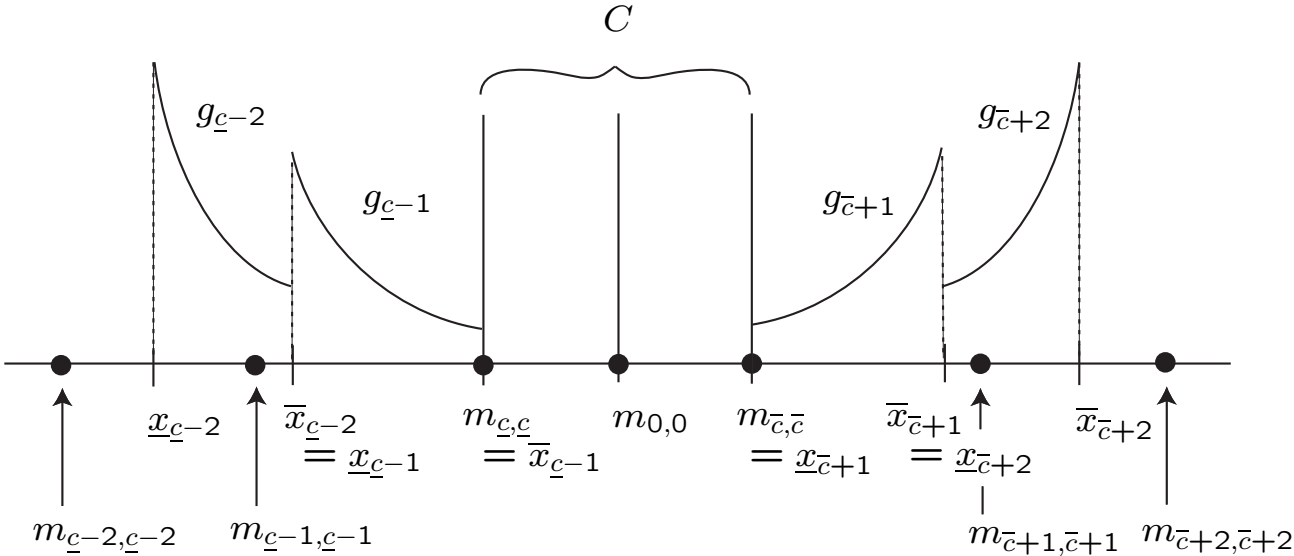
- * For all $i \in C$, candidates locate at $m_{i,i}$.
- * For all $i > C$, candidates mix according to the increasing, convex density

$$g_i(x) = \frac{\Phi_i}{2} \sqrt{\frac{m_{i,i} - \underline{x}_i}{(m_{i,i} - x)^3}}.$$

- * Candidates with extreme signals moderate relative to pure strategy equilibrium locations: $\bar{x}_i < m_{i,i}$.
- * Supports are adjacent: for all $i > C$, $\bar{x}_{i-1} = \underline{x}_i$.

Necessary conditions (cont.)

- A picture of equilibrium strategies:

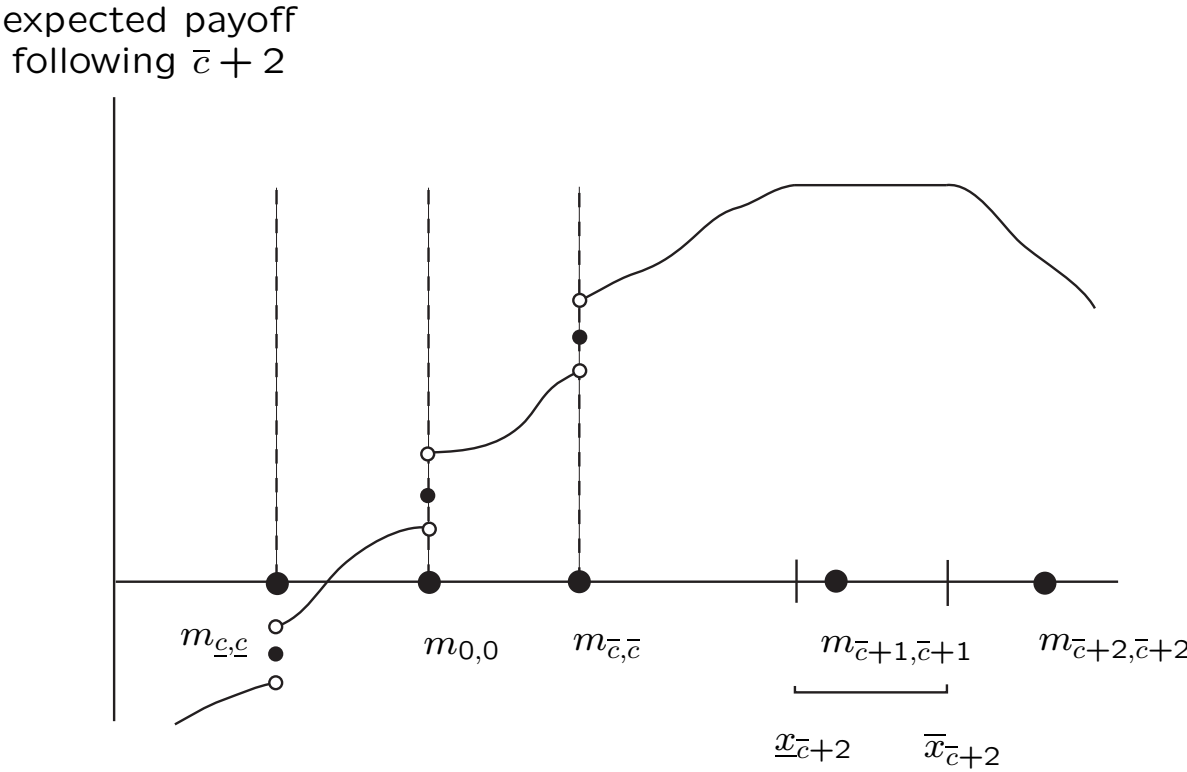


Sufficient conditions

- (A4) Self-likelihood: For all signals i and k , $P(k|k) \geq P(k|i)$.
- **Theorem 3.** Under (A2)–(A4), if one candidate adopts the strategy specified in Theorem 2, then following each signal i , the other candidate's payoff is weakly single-peaked in his location x_i and maximized by all $x_i \in [\underline{x}_i, \bar{x}_i]$.
- **Corollary 1.** Under (A2)–(A4), there exists a unique ordered equilibrium, as specified above.
- Note: These are maxmin strategies.

Sufficient conditions (cont.)

- Picture of expected payoffs:



Implications

- The expected location following signal $i > C$ is

$$E[x_i] = \frac{\Phi_i}{1 + \Phi_i} x_i + \frac{1}{1 + \Phi_i} m_{i,i}.$$

- Increasing the conditional medians $m_{i,i}$ for $i > C$ leads to a first order stochastic increase in the equilibrium distributions.
- Reducing

$$\Phi_i = \frac{\sum_{j:j \leq i} P(j|i) - \frac{1}{2}}{P(i|i)}$$

for $i > C$ also leads to a first order stochastic increase.

Voter welfare

- Assume the distribution of ideal points in the electorate is fixed, up to parameter μ .

- Quadratic utility: For each voter v ,

$$u_v(z) = -(\mu + \delta_v - z)^2.$$

- Symmetry around zero:

- * $i = -K, \dots, -1, 0, 1, \dots, K$

- * $b = -B, \dots, -1, 0, 1, \dots, B$

- * $P(b) = P(-b)$

- * $P(i, j|b) = P(-i, -j| -b)$.

Voter welfare (cont.)

- Suppose a social planner can require the candidates to locate according to a strategy $X = (x_i)$, where X is monotone in signal.
- Let $W_\delta(X)$ denote the ex ante welfare for the voter who is δ from μ when candidates locate according to X .
- Say that X satisfies **zero-symmetry** if $x_i = -x_{-i}$ for all signals i .

Voter welfare (cont.)

- **Proposition 2.** Voter v 's welfare is a fixed amount less than the welfare of the median voter:

$$W_{\delta_v}(X) = -\delta_v^2 + W_0(X)$$

for all zero-symmetric strategies.

- Thus, we focus on the median voter's welfare W without loss of generality.
- **Proposition 3.** The welfare function W is strictly concave.

Optimal separation

- Separation by candidates can be socially beneficial, because it leads to more choice for voters.

- **Theorem 5.** Assume (A1). For all signals $i > 0$,

$$\frac{\partial W}{\partial x_i}(M) > 0$$

if:

- * $P(\cdot|i)$ exhibits “inertia,” i.e.,

$$P(i - h|i) > P(i + h|i),$$

- * there is sufficient uncertainty about the location of the median voter.
- The result always holds in the model with two or three signals.

Optimal separation (cont.)

- Define the unique social welfare optimum X^* by

$$W(X^*) = \max W(X).$$

- (A5) Strong single-peakedness: For all distinct signals $i, j > 0$,

$$\frac{P(j|i)}{P(-j|i)} > \frac{3m_{i,i} + m_{j,j}}{|m_{i,i} - m_{j,j}|}.$$

- This condition always holds in the model with two or three signals.

Optimal separation (cont.)

- **Theorem 6.** Assume (A5) and for all signals $i > 0$,

$$\frac{\partial W}{\partial x_i}(M) > 0.$$

Then the socially optimal platforms of the candidates are more extreme than the conditional medians, i.e., $x_i^* > m_{i,i}$ for all $i > 0$.

- **Corollary 2.** Adding (A2) and (A3), the candidates locate too moderately in equilibrium relative to the social optimum with probability one.

Effects of signal correlation

- Assume that with probability q both candidates receive the same signal drawn from $P(\cdot|b)$, and with probability $1 - q$ they receive conditionally independent signals drawn from $P(\cdot|b)$.
- (A6) Strong signals: For all signals $i > 0$, $m_{i,i} > m_{i,-i}$.
- This always holds in the model with two or three signals.

Effects of signal correlation (cont.)

- **Theorem 7.** Assume (A6) and for all signals $i > 0$,

$$\frac{\partial W}{\partial x_i}(M) > 0.$$

When the pure strategy equilibrium exists, higher correlation q leads to lower voter welfare:

$$\frac{dW}{dq}(M) < 0.$$

- Thus, private polling is preferred to public.
- **Conjecture.** The same holds for the mixed strategy equilibrium.

Effects of signal precision

- The effects of signal precision on welfare are more subtle: Greater precision allows candidates to better target the median voter, but they also are more likely to get the same signal (and locate together).
- Consider the two-signal model: $b = -1, 1$ and $i = -1, 1$. Denote q as above, and let p be the probability of a correct signal.
- Then the socially optimal level of precision is above zero and below one.
- If signal precision depends positively on campaign spending, this suggests one argument for spending caps in electoral campaigns.

Conclusion

- Candidates separate because they condition their platforms on private information.
- In pure strategy equilibrium, candidates tend to overshoot the median voter.
- In mixed strategy equilibrium, the same is true for candidates with moderate signals. Candidates with more extreme signals moderate their platforms.
- We give theoretical foundations for the claim that candidates do not separate enough.
- Correlation is bad, because it reduces separation.
- Some precision is good, but not too much.