

PSC 403 MATHEMATICAL MODELING

Fall, 2000  
MW 10:00-11:50  
Harkness 329

Professor: John Duggan  
Office: Harkness 111A, x34999

My goal in this course is to expose you to several mathematical models important in social science, and at the same time to provide you with the mathematical knowledge needed to understand them, to use them, and (if it suits you) to surpass them in your own research in political science. The course will emphasize rigorous logical and deductive reasoning — this skill will prove valuable, even to the student primarily interested in empirical analysis rather than modelling.

Homeworks, possibly a midterm, and definitely a final will be designed to help you develop your mathematical skills. There are two textbooks for the course.

- Simon and Blume (SB), *Mathematics for Economists*
- Binmore (B), *Mathematical Analysis*

I will supplement these by distributing my lecture notes.

An outline of the topics to be covered is as follows. Next to each, I list suggested readings from the texts. It will be clear from the selections of readings that the authors, especially Simon and Blume, organized their books differently than I'm organizing the course. Later chapters, say Simon and Blume's Chapter 22 on "Economic Applications," won't be easily accessible the first time through.

1. logic, sets, relations (SB A1; B 1)
  - finite sets, real numbers, vectors, weak orders, partial orders, individual preference, Pareto dominance, majority preference, maximal elements, Pareto optimality
2. functions (SB 2.1, 2.2, 5.1-5.4, 13.1-13.3, 13.5, A2; B 7, 14, 16)
  - linear, affine, quadratic, polynomial, quasi-linear, Cobb-Douglas, logs, exponentials, trigonometric, graphs, level sets, upper and lower sections, maxima and minima

3. basic models (SB 22)
  - finite choice, tournaments, spatial model, cost-benefit analysis, the firm, the consumer, markets, exchange economies, two-person bargaining, random variables, expected utility, matrix games, linear estimation
4. geometry of  $\mathfrak{R}^n$  (SB 10, 21.1-21.3; B 18.1-18.20)
  - vector addition, scalar multiplication, lines, convex sets, simplexes, linear functions, hyperplanes, quasi-concavity, concavity, inner products, orthogonality, projections, gradients, norms
5. matrices (SB 6-9, 16.1, 16.2, 26; B 18.22-18.25)
  - systems of linear equations, Leontief production, etc., transposes, inverses, Gauss-Jordan elimination, determinants, Cramer's rule, definiteness, quadratic forms
6. real analysis (B 2, 4-9; SB 12, 13.4, 29)
  - real numbers, suprema, infima, completeness, sequences, series, closed sets, open sets, compact sets, continuous functions
7. calculus (B 10-12; SB 2.3, 2.4, 3.1-3.4, 4, 5.5, 5.6, 14, 15, 30)
  - derivatives, mean value theorem, directional derivatives, partial derivatives, marginal rate of substitution, chain rule, Taylor's theorem, gradients, cross partials, Hessians, implicit function theorem
8. unconstrained optimization (SB3.5, 3.6, 17, 21.5)
  - first and second order necessary conditions and sufficient conditions for local maximizers, global maximizers, cost-benefit analysis, profit maximizing firms, the envelope theorem, comparative statics
9. constrained optimization (SB 18, 19, 21.5)
  - Lagrange's method, Kuhn-Tucker method, consumer choice, social planning, campaign spending, Nash bargaining, agenda setting, the envelope theorem, comparative statics

10. integration (B 13; SB A4)

- Riemann integral, fundamental theorem of calculus, integration by parts and substitution, consumer's (s') surplus, expected value, variance, expected utility and risk aversion, mean-variance analysis

If we get to this point, you will have learned more than enough about mathematical modeling to succeed in your studies and future research, or at least enough to learn what you'll need to know. If we somehow find ourselves with extra time, or you want to pursue studies in mathematics after the semester, here are a few more topics worth covering.

11. geometry of  $\mathfrak{R}^n$ , continued (SB 11, 28)

- linear combinations and spans, linear independence, bases, subspaces, dimension, Gram-Schmidt orthogonalization process, OLS

12. matrices, continued (SB 23, 27)

- linear operators, rank, range and null spaces, fundamental theorem of linear algebra, eigenvalues, eigenvectors

13. miscellaneous topics (SB 24, 25, and elsewhere...)

- separating hyperplanes, correspondences, theorem of maximum, fixed point theorems, Lebesgue integration, differential equations