Assignment 3 (Due Wednesday, October 26)

1. Suppose that $X \sim N(\mu, \Omega)$. Show that $Y = \Omega^{-1/2}(X - \mu)$ has the standard normal distribution (Hint: Recall that $\det(AB) = \det(A)\det(B)$, which also implies that $\det(A^{-1}) = \det(A)^{-1}$).

2. Suppose that X is a random vector. Show that,

(a) $Var(X) = E[XX^T] - \mu\mu^T$

(b) $Var(AX + b) = AVar(X)A^T$

(c) $Cov(X, Y) = E[XY^T] - \mu_X \mu_Y^T$

3. Let X be a random variable that has the $N(1, 3^2)$ distribution with probability 0.3 and has the $U(1, 2)$ distribution with probability 0.7.

(a) Characterize the cdf of X.

(b) What are $E[X]$ and $Var(X)$?

(c) What is the characteristic function of X?

4. Suppose that Y has the multinomial distribution with N trials and probability vector π.

(a) What are the mean and variance of Y_k?

(b) What is the correlation between Y_j and Y_k for $j \neq k$?

(Hint: for both parts, use the fact that Y can be written as $Y_k = \sum_{n=1}^{N} X_{n,k}$ where X_n are iid discrete random variables with pmf $p_X(x) = \pi_x$.